Textbooks and Notations

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Textbooks and grading

- Textbooks

John Cardy	*"Scaling and renomalization in statistical physics"* (Cambridge)
Hidetoshi Nishimori	*"Souten'i, rinkaigenshou no toukeibutsurigaku"* (Baifukan) in Japanese

- Grading

  Grading is based on the reports on the excercize problems presented at lectures (about 3-4 times during the whole course)

Equality

$$
\begin{array}{ll}
x = y & \text{(exactly equal)} \\
x \equiv y & \text{(definition)} \\
x \approx y & \text{(approximately equal)} \\
x \sim y & \text{(equal apart from a dimensionless constant)} \\
x \propto y & \text{(equal apart from a constant)}
\end{array}
$$

Inverse temperature $\beta \equiv 1/k_{\mathrm{B}}T$

In most cases, the inverse temperature is included in the definition of the Hamiltonian $\mathcal{H}$. (Sometimes, k_{B} may be omitted, assuming that the unit of the temperature is taken appropriately.)

Field variables

A function of field variables, such as the Ising model Hamiltonian that depends on Ising spins, e.g., $S_1, S_2, \cdots$, may be expressed as

$$\begin{aligned}
&\mathcal{H}(S_1, S_2, \cdots, S_N) \\
&\qquad = \mathcal{H}(\{S_i | i \in \Omega\}) \quad (\Omega = \{1, 2, \cdots, N\} \\
&\qquad = \mathcal{H}(\{S_i\}_{i\in\Omega})
\end{aligned}$$

When the definition of the space Ω is clear from the context, or when it does not have to be specified, we may drop it and use the simpler symbols such as

$$\mathcal{H}(\{S_i\}), \quad \mathcal{H}(\boldsymbol{S}), \quad \mathcal{H}(S), \quad \cdots$$

for the same meaning.

Tr (trace) and $\int$ (integral) for functional integrals

For functional integrals with respect to the field variables, we often use the both symbols for the same meaning, i.e.,

$$\operatorname*{Tr}_{\phi} f[\phi(\boldsymbol{x})] \equiv \int D\phi(\boldsymbol{x})\, f[\phi(\boldsymbol{x})]$$

Fourier transformation and Greens' functions

$$a = \text{(lattice constant)}, \quad L = \text{(system size)}, \quad N \equiv \frac{L^d}{a^d} = (\#\text{ of sites})$$

$$\tilde{\phi}_{\boldsymbol{k}} = \int_0^L d^d\boldsymbol{r}\, e^{-i\boldsymbol{k}\boldsymbol{r}} \phi_{\boldsymbol{r}} = a^d \sum_{\boldsymbol{r}} e^{-i\boldsymbol{k}\boldsymbol{r}} \phi_{\boldsymbol{r}}$$

$$\phi_{\boldsymbol{r}} = \int_{-\pi/a}^{\pi/a} \frac{d^d\boldsymbol{k}}{(2\pi)^d} e^{i\boldsymbol{k}\boldsymbol{r}} \tilde{\phi}_{\boldsymbol{k}} = L^{-d} \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\boldsymbol{r}} \tilde{\phi}_{\boldsymbol{k}}$$

The tilde ˜ is often dropped when there is no fear of confusion, e.g.,

$$G(\boldsymbol{r}', \boldsymbol{r}) \equiv \langle \phi_{\boldsymbol{r}'} \phi_{\boldsymbol{r}} \rangle, \quad G_{\boldsymbol{k}',\boldsymbol{k}} \equiv L^{-d} \langle \phi_{\boldsymbol{k}'} \phi_{\boldsymbol{k}} \rangle$$

For translationally and rotationally symmetric case,

$$G(\boldsymbol{r}', \boldsymbol{r}) = G(|\boldsymbol{r}' - \boldsymbol{r}|), \quad G_{\boldsymbol{k}',\boldsymbol{k}} = \delta_{\boldsymbol{k}'+\boldsymbol{k},\boldsymbol{0}} G_{|\boldsymbol{k}|}, \quad G_{|\boldsymbol{k}|} \equiv L^{-d} \langle |\phi_{\boldsymbol{k}}|^2 \rangle$$

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$\acute{X}$(acute) and X'(prime)

We use both for the same meaning, because the position of the mark for the prime sometimes interferes with other superscripts and look messy.

“Rank”

In conventional mathematical terminology, the word “rank” can mean two things: the number if indices of a tensor and the number of linearly independent column/row vectors of a matrix, or more generally, the number of non-zero singular values of a tensor. When we talk about tensors, this dual meaning causes confusion. To avoid it, we use the word “rank” only for the latter. For the number of indices of a tensor, we use “degree”. For example, a third degree tensor, or a tensor of degree 3, is a tensor with three indices, and a rank-n matrix is a matrix with n independent column/row vectors.

Normal order product

We use the symbol $[\![\cdots]\!]$ for the normal-order product. In text books, colons $(:\cdots:)$ are more often used. (There is no reason. Just a matter of taste.)