Notations and Conventions

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Equality

$$
\begin{aligned}
x = y &\quad \text{(exactly equal)} \\
x \approx y &\quad \text{(approximately equal)} \\
x \sim y &\quad \text{(equal apart from a dimensionless constant)} \\
x \propto y &\quad \text{(equal apart from a constant)}
\end{aligned}
$$

Inverse temperature $\beta \equiv 1/k_{\mathrm{B}}T$

In most cases, the inverse temperature is included in the definition of the Hamiltonian $\mathcal{H}$.

Field variables

A function of field variables, such as the Ising model Hamiltonian that depends on Ising spins, e.g., $S_1, S_2, \cdots$, may be expressed as

$$
\begin{aligned}
&\mathcal{H}(S_1, S_2, \cdots, S_N) \\
&\qquad = \mathcal{H}(\{S_i | i \in \Omega\}) \quad (\Omega = \{1, 2, \cdots, N\} \\
&\qquad = \mathcal{H}(\{S_i\}_{i\in\Omega})
\end{aligned}
$$

When the definition of the space Ω is clear from the context, or when it does not have to be specified, we may drop it and use the simpler symbols such as

$$
\mathcal{H}(\{S_i\}), \quad \mathcal{H}(\mathbf{S}), \quad \mathcal{H}(S), \quad \cdots
$$

Tr (trace) and $\int$ (integral) for functional integrals

For functional integrals with respect to the field variables, we often use the both symbols for the same meaning, i.e.,

$$\mathop{\mathrm{Tr}}_{\phi} f[\phi(\mathbf{x})] \equiv \int D\phi(\mathbf{x})\, f[\phi(\mathbf{x})]$$

Fourier transformation and Greens' functions

$$a = (\text{lattice constant}), \quad L = (\text{system size}), \quad N \equiv \frac{L^d}{a^d} = (\#\text{ of sites})$$

$$\tilde{\phi}_{\mathbf{k}} = \int_0^L d^d\mathbf{r}\, e^{-i\mathbf{kr}}\phi_{\mathbf{r}} = a^d \sum_{\mathbf{r}} e^{-i\mathbf{kr}}\phi_{\mathbf{r}}$$

$$\phi_{\mathbf{r}} = \int_{-\pi/a}^{\pi/a} \frac{d^d\mathbf{k}}{(2\pi)^d}\, e^{i\mathbf{kr}}\tilde{\phi}_{\mathbf{k}} = L^{-d}\sum_{\mathbf{k}} e^{i\mathbf{kr}}\tilde{\phi}_{\mathbf{k}}$$

The tilde ˜ is often dropped when there is no fear of confusion.

$$G(\mathbf{r}', \mathbf{r}) \equiv \langle \phi_{\mathbf{r}'}\phi_{\mathbf{r}} \rangle, \quad G_{\mathbf{k}',\mathbf{k}} \equiv L^{-d}\langle \phi_{\mathbf{k}'}\phi_{\mathbf{k}} \rangle$$

For translationally and rotationally symmetric case,

$$G(\mathbf{r}', \mathbf{r}) = G(|\mathbf{r}' - \mathbf{r}|), \quad G_{\mathbf{k}',\mathbf{k}} = \delta_{\mathbf{k}'+\mathbf{k},\mathbf{0}}G_{|\mathbf{k}|}, \quad G_{|\mathbf{k}|} \equiv L^{-d}\langle |\phi_{\mathbf{k}}|^2 \rangle$$

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$\acute{X}$(acute) and X'(prime)

We use both for the same meaning, because the position of the mark for the prime sometimes interferes with other superscripts and look messy.

“Rank”

The word “rank” can mean two things: the number if indices of a tensor and the number of linearly independent column/row vectors of a matrix. To avoid confusion, we use the word only for the latter. For the number of indices of a tensor, we use “degree”. So, a third degree tensor is a tensor with three indices and a rank-n matrix is a matrix with n independent column/row vectors.

Normal order product

We use the symbol $[\![\cdots]\!]$ for the normal-order product. In text books, colons $(: \cdots :)$ are more often used. (There is no reason. Just a matter of taste.)