# Bose-Einstein Condensation of Tripletons

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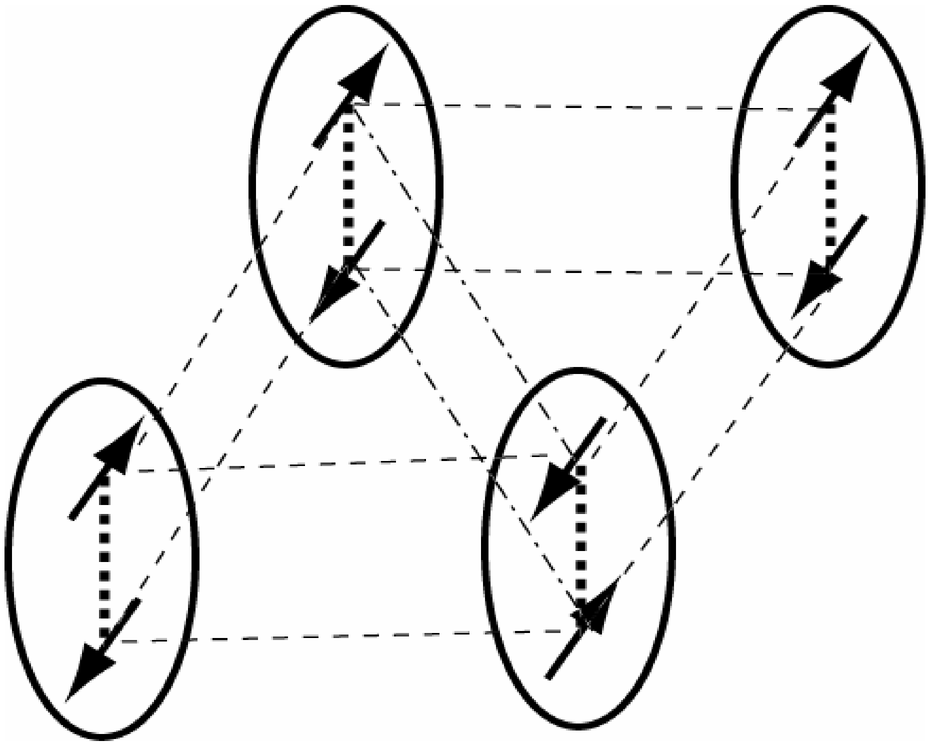
# Spin Dimer Systems



# Spin Dimer Materials

$SrCu_2(BO_3)_2$ … super solid of tripletons
Kageyama, et al., PRL82 (1999); PRL84 (2000); Kodama, et al., Science 298 (2002).

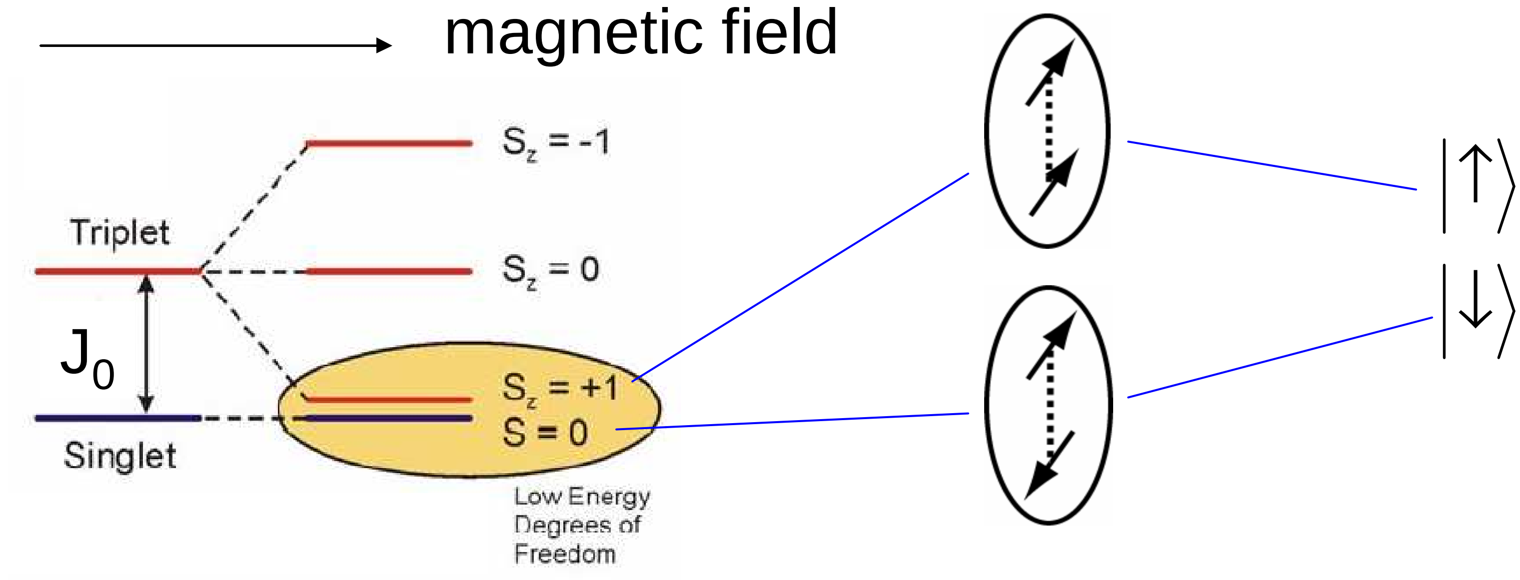
$TlCuCl_3$ … super fluid
Oosawa, et al., J.Phys.:Cond. Mat. 11 (1999); Nikuni, et al., PRL 84 (2000); Ruegg, et al., Nature 423 (2003).

$KCuCl_3$ … super fluid
Cavadini, et al., PRB65 (2002); Oosawa, et al., PRB66 (2002).

$BaCuSi_2O_6$ … super fluid (specific heat and magnetization)
Jaime et al, PRL93 (2004).

# Effective Spins (Tripletons)

- Tachiki-Yamada J.Phys.Soc.Jpn. 28 (1970) 1413.




$$\left|\uparrow\right\rangle \equiv \left|\sigma^z = -\frac{1}{2}\right\rangle \equiv \left|S = 0\right\rangle$$

$$\left|\downarrow\right\rangle \equiv \left|\sigma^z = +\frac{1}{2}\right\rangle \equiv \left|S = 1, S^z = 1\right\rangle$$

# Effective Spin Model (XXZ model)

- Tachiki-Yamada J.Phys.Soc.Jpn. 28 (1970) 1413.

$$H_{\text{original}} = J\sum_{i}\mathbf{S}_{i,1}\cdot\mathbf{S}_{i,2} + J'\sum_{(ij)}\sum_{\alpha=1,2}\mathbf{S}_{i,\alpha}\cdot\mathbf{S}_{j,\alpha} - g\mu_{\text{B}}H_0\sum_{i,\alpha}S^z_{i,\alpha}$$

$$\Downarrow$$

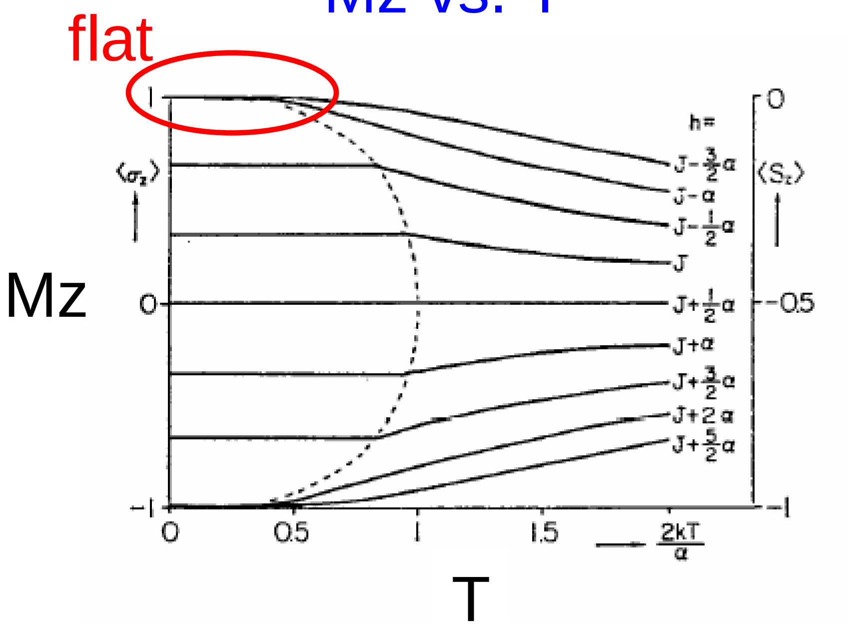
$$H_{\text{effective}} = J'\sum_{(ij)\perp\text{z-axis}}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y + \frac{1}{2}\sigma_i^z\sigma_j^z\right) - H\sum_i\sigma_i^z$$

$$H \equiv g\mu_{\text{B}}H_0 - J - \frac{z}{4}J'$$

# Mean-Field Solution

$$H_{\mathrm{mf}} = J' \sum_{(ij)\perp \text{z-axis}} \left( \sigma_i^x \langle \sigma_j^x \rangle + \sigma_i^y \langle \sigma_j^y \rangle + \frac{1}{2} \sigma_i^z \langle \sigma_j^z \rangle \right) - H \sum_i \sigma_i^z$$

Mz vs. T




T-H Phase Diagram

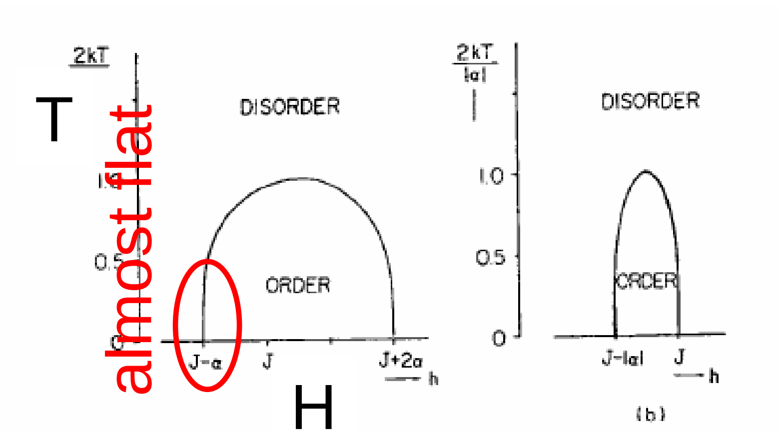
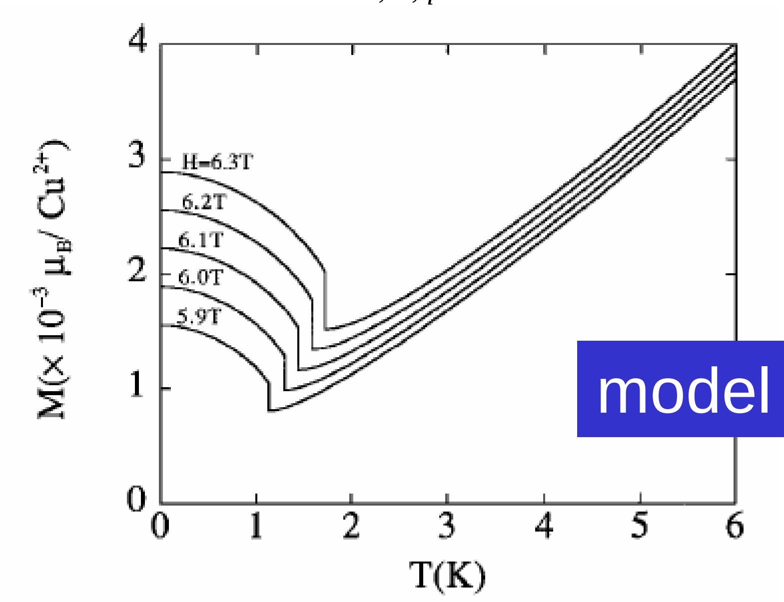



Fig. 17. The field dependence of the critical temperature $T_c$, (a) for the antiferromagnetic case, and (b) for the ferromagnetic case.

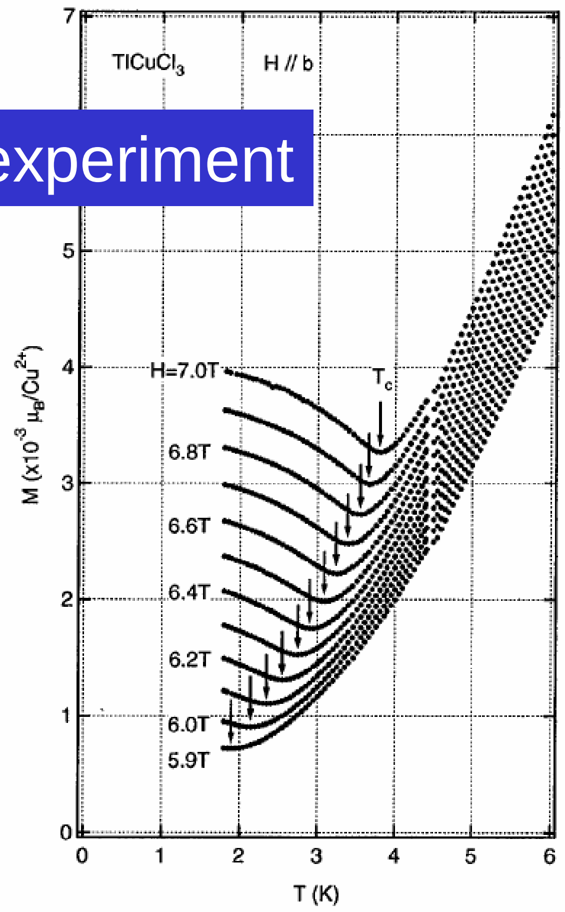
# Qualitative Description by Tripleton BEQ

**Hartree-Fock** approximation: T.Nikuni, et al. PRL84 (2000) 5868
(See also Misguich and Oshikawa cond-mat/0405422.)

$$H = \sum_k \left( \frac{\hbar^2 k^2}{2m} - \mu \right) a_k^+ a_k + \frac{v_0}{2} \sum_{k,k',q} a_{k+q}^+ a_{k'-q}^+ a_k a_{k'}$$



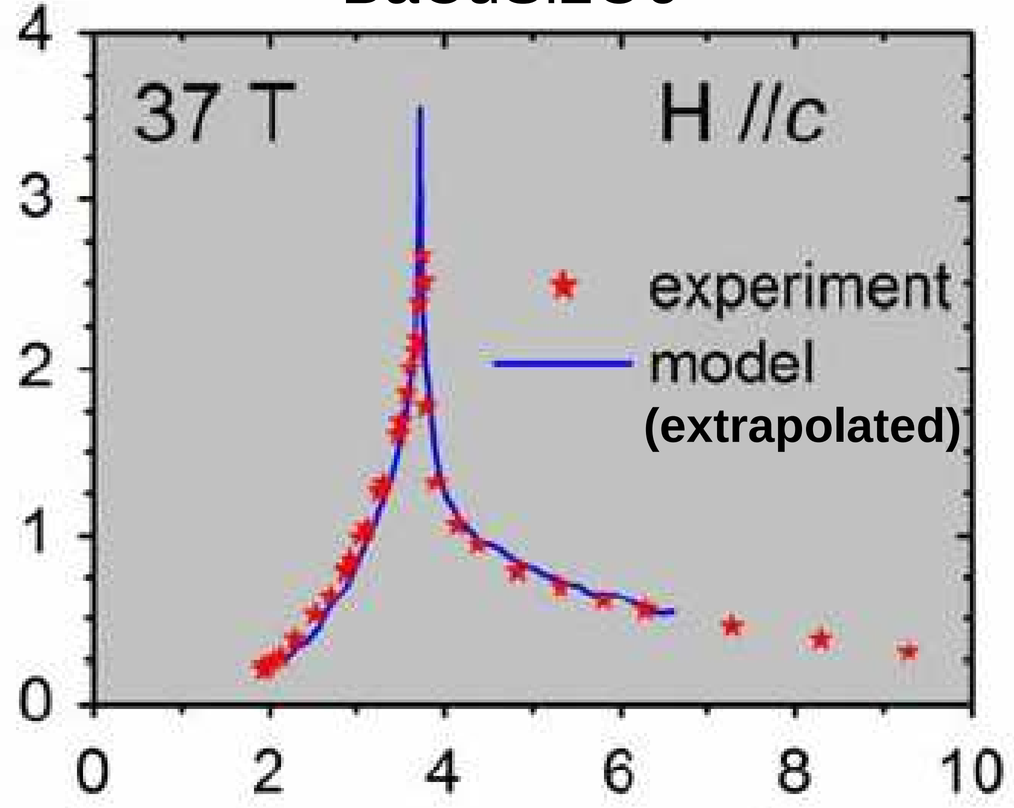

model (HF)




experiment

# Specific Heat

BaCuSi2O6

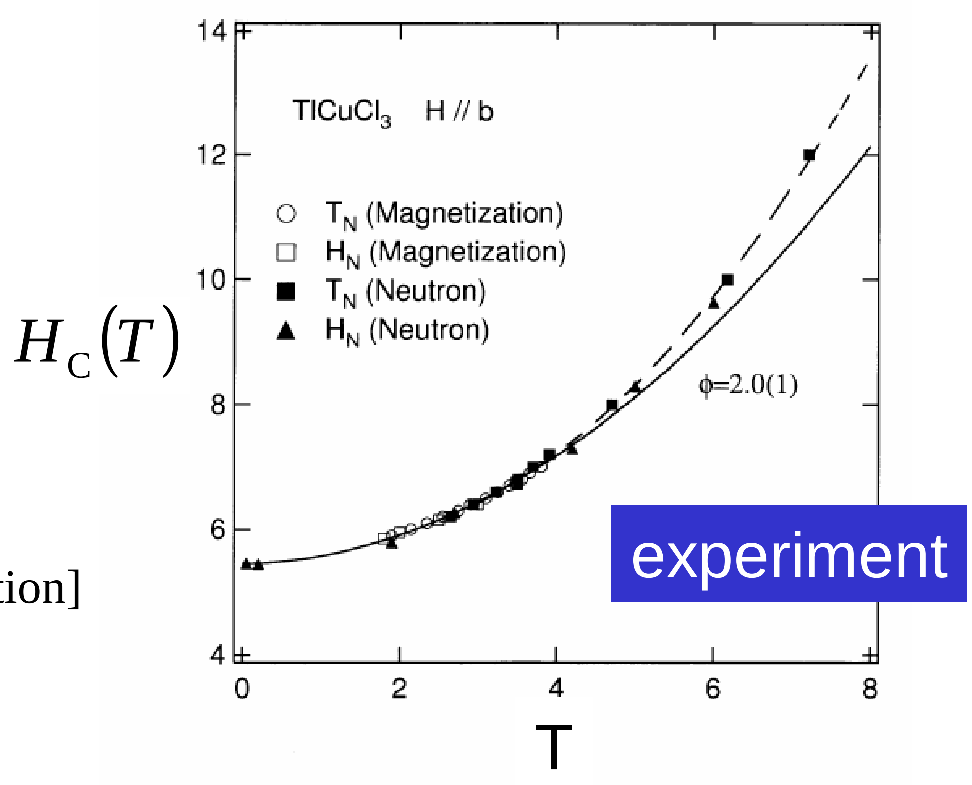



Jaime et al, PRL93 (2004).

# Remaining Disagreement

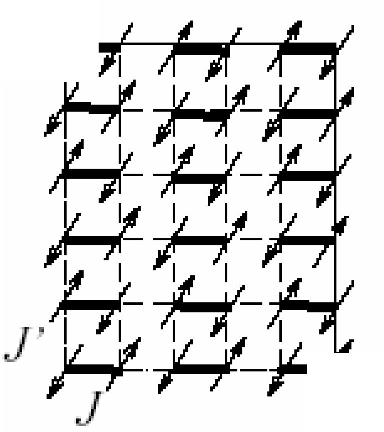
$$H_C(T) - H_C(0) \propto T^{\phi}$$

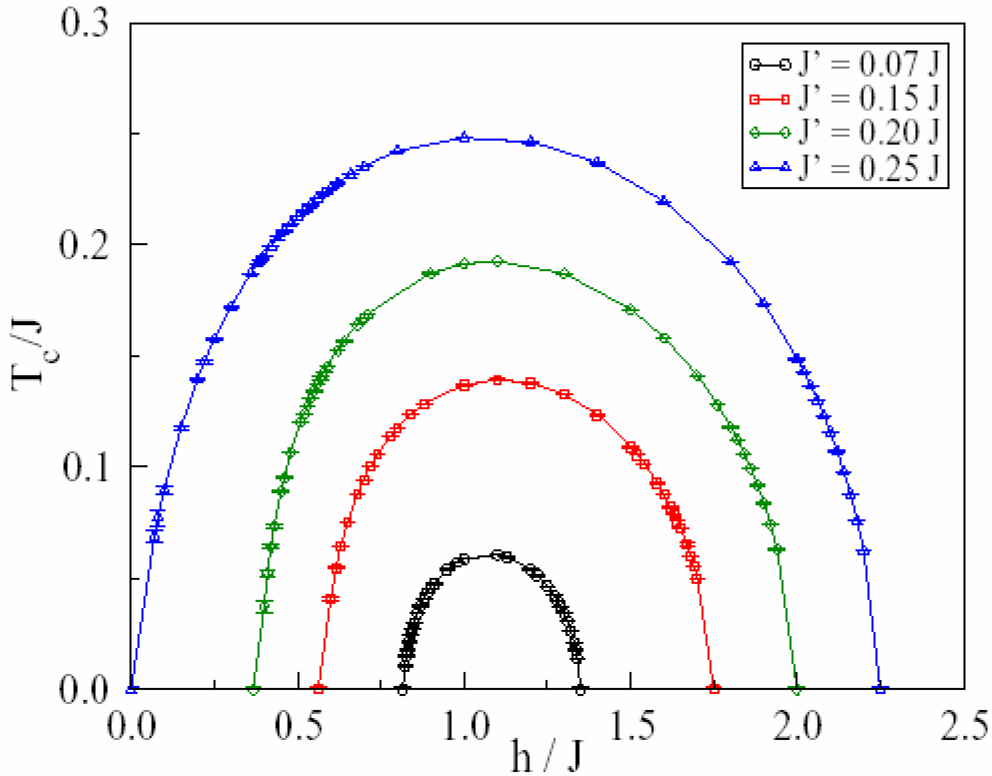
$$\phi = \begin{cases} 2.0(1) & \text{[Experiment]} \\ 1.5 & \text{[MF approximation]} \end{cases}$$




Tanaka et al: J.Phys.Soc.Jpn.70 (2001)

# Recent Simulation of XXZ model with realistic coupling constants




**$\phi$=1.6～2.0**

It approaches 1.5 as fitting range is narrowed.

Nohadani, Wessel, Normand, Haas: Phys. Rev. B 69, 220402 (2004)

# Field Theoretical Description of QCP

corresponds to magnetic field

$$Z = \int D\psi D\psi^* \exp\left( - \int_0^{1/T} d\tau \int d^d x\, L \right)$$

$$L = \psi^* \frac{\partial \psi}{\partial \tau} + \frac{1}{2m} \left|\nabla \psi\right|^2 - h \,\mathrm{Re}\,\psi + t \left|\psi\right|^2 + \frac{u}{2}\left|\psi\right|^4$$

Quantum Critical Point (QCP): $h = t = u = 0$

A simple dimensional analysis yields

$$\psi \to b^{d/2}\psi, \quad L \to b^{-1}L, \quad \beta \to b^{-2}\beta \qquad (z = 2)$$

$$h \to b^{2+d/2}, \; t \to b^2 t$$

$$u \to b^{2-d} u \qquad \text{(dangerous irrelevant)}$$

d=3 is above the upper critical dimension.

# Size Dependence of $\chi$

$$\chi(t,u,\beta,L) = L^{-d}\beta^{-1}\frac{\partial^2}{\partial h^2}\Phi(h,t,u,\beta,L)\bigg|_{h\to 0}$$

$$= L^{-d}\beta^{-1}\frac{\partial^2}{\partial h^2}\tilde{\Phi}\left(hL^{2+d/2}, tL^2, uL^{2-d}, \beta L^{-2}\right)\bigg|_{h\to 0}$$

$$= L^2\ \tilde{\chi}\left(tL^2, uL^{2-d}, \beta L^{-2}\right)$$

$\tilde{\chi}(t',u',c)$: scaling function

At $t'=0$, in the limit $u'\to 0$ with fixed $c$

$$\tilde{\chi}(0,u',c) \propto 1\big/\sqrt{u'}$$

This yields

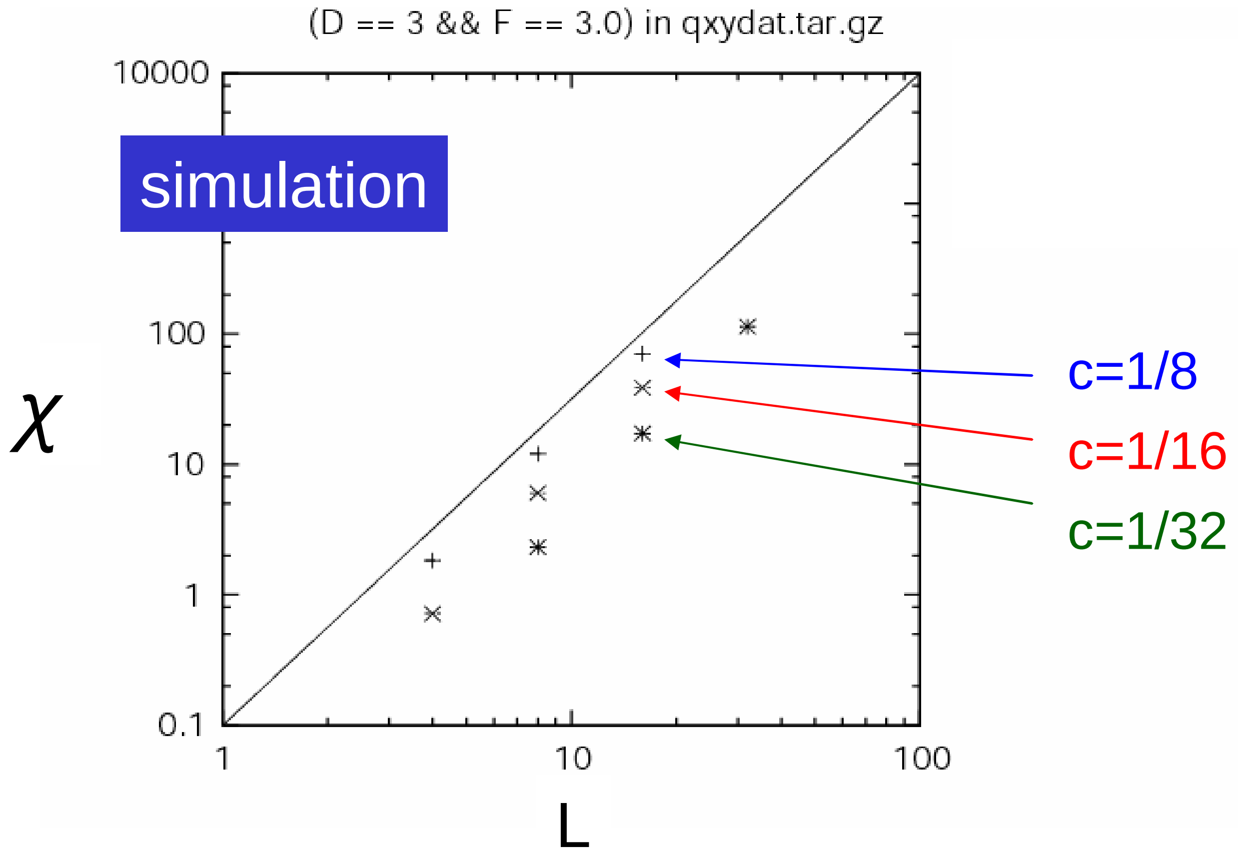
$$\chi(0,u,\beta=cL^2,L) = L^2\Big/\sqrt{uL^{2-d}} \propto L^{1+d/2}$$

# 3D XY model

$$H = J\sum_{(ij)}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y\right) - H\sum_i \sigma_i^z$$

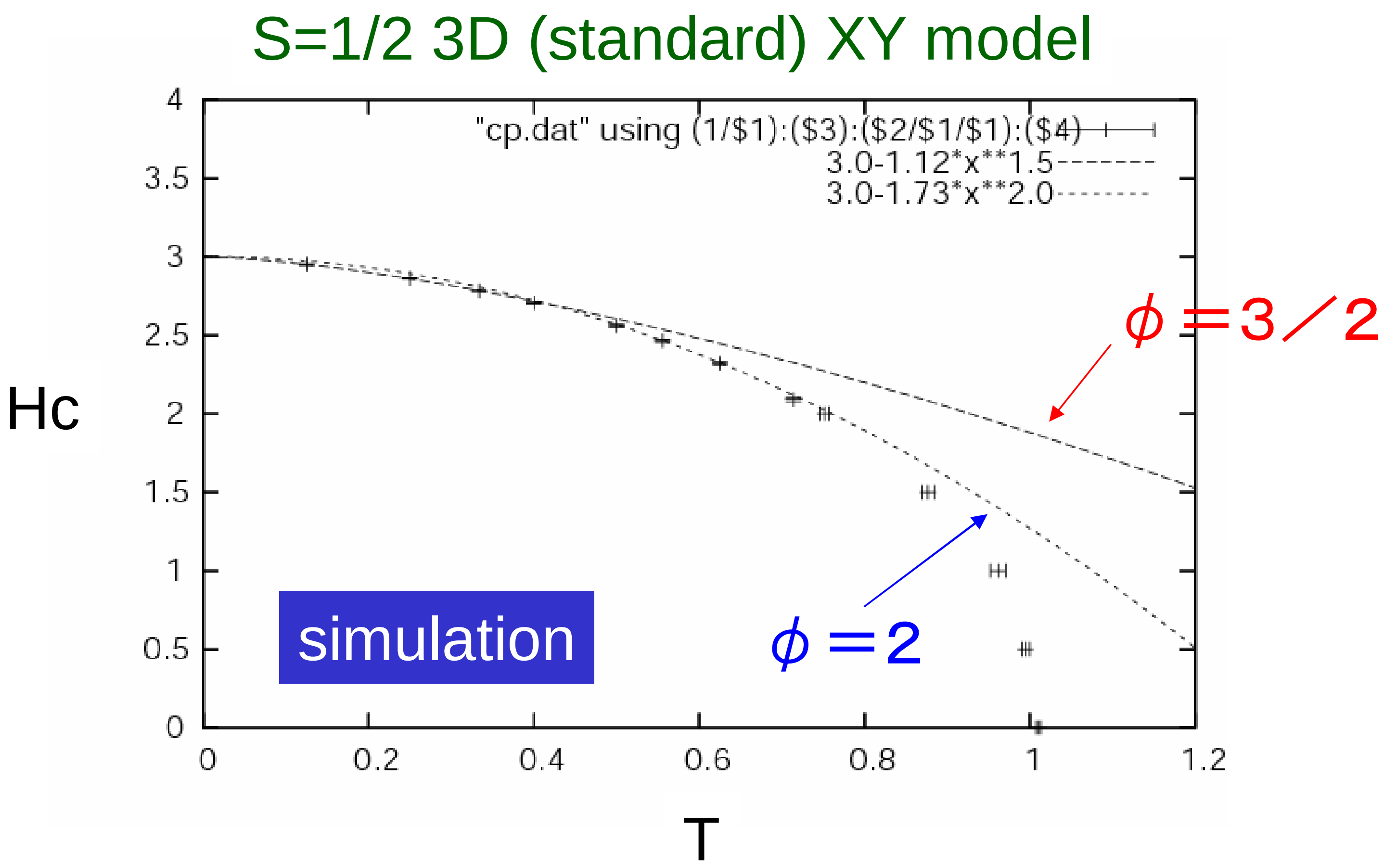
**… simple, but believed to show the same critical behavior as the realistic model at a finite temperature and at zero temperature**

# Size Dependence of $\chi$

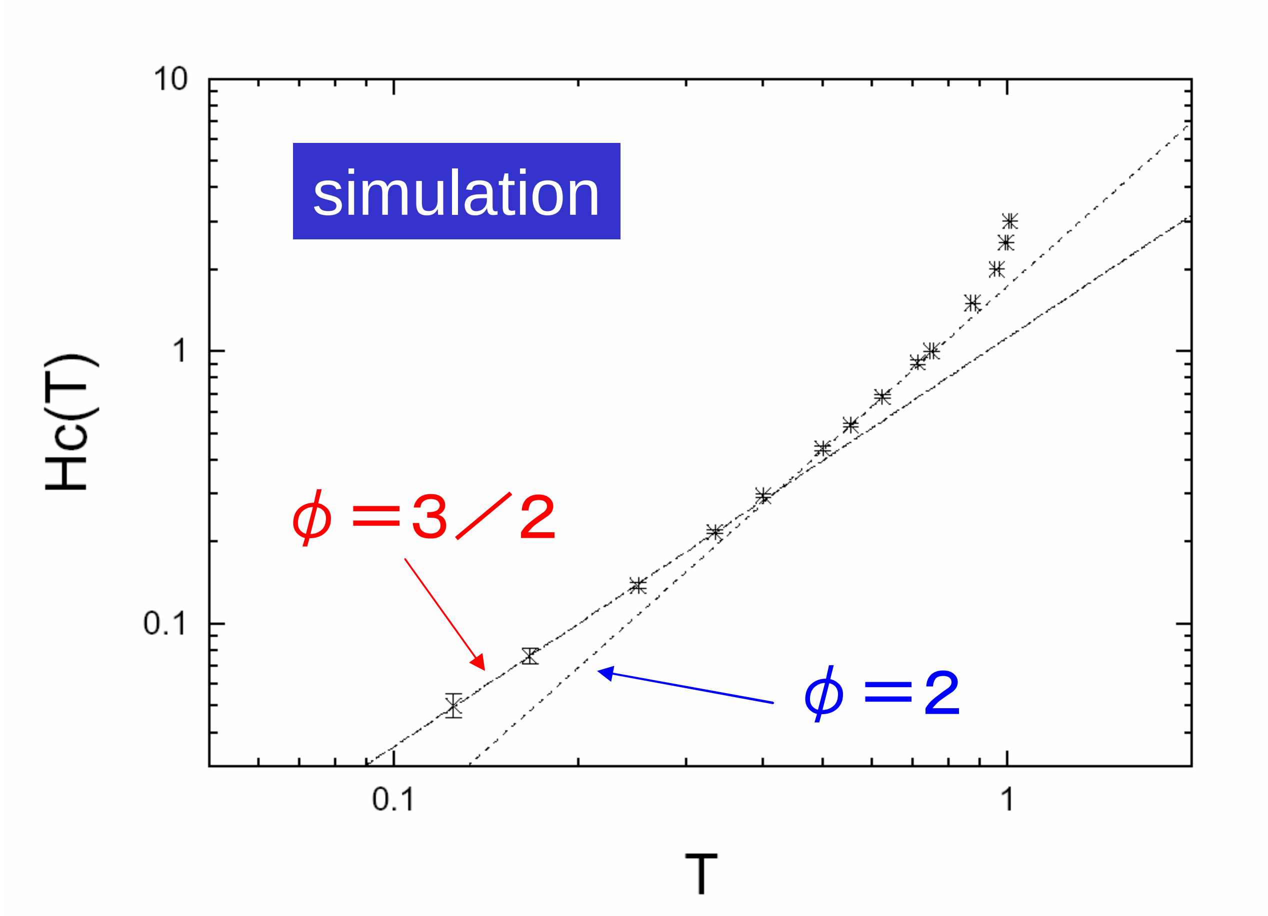



$$\chi_{xx}\left(H = H_C, \beta = cL^2, L\right) \propto L^{2.5}$$

# Critical Field



# Critical Field




φ≒2 describes an only transient behavior.
The correct value seems to be φ＝3／2.

# Summary

Mean-field scaling is exact for the QCP.

Experiments may be performed
only in a transient regime.