論文紹介:

“Corrections to Scaling are Large for Droplets in Two-Dimensional Spin Glasses”

A.K.Hartmann and M.A.Moore

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川島直輝 （都立大理）

２００３年７月２３日　千葉大学　セミナー

Two-Dimensional Spin Glass

$$H = -\sum_{(ij)} J_{ij} S_i S_j \qquad P(J_{ij}) = \frac{1}{\sqrt{\pi J}} e^{-J_{ij}^2 / J^2}$$

Widely accepted hypothesis …

- No phase transition at finite temperature.
- T=0 is critical.

One-Parameter Scaling

$$\log Z_{\text{singular}}(T,H,L) = f\left(TL^{y}, HL^{y_h}\right)$$

$$\Rightarrow \quad M(T,H,L) = L^{y_h - y}\, \tilde{m}\left(TL^{y}, HL^{y_h}\right)$$

However, at T=0, the state is frozen to be the ground state, which is a complete random assignment of +1/-1.

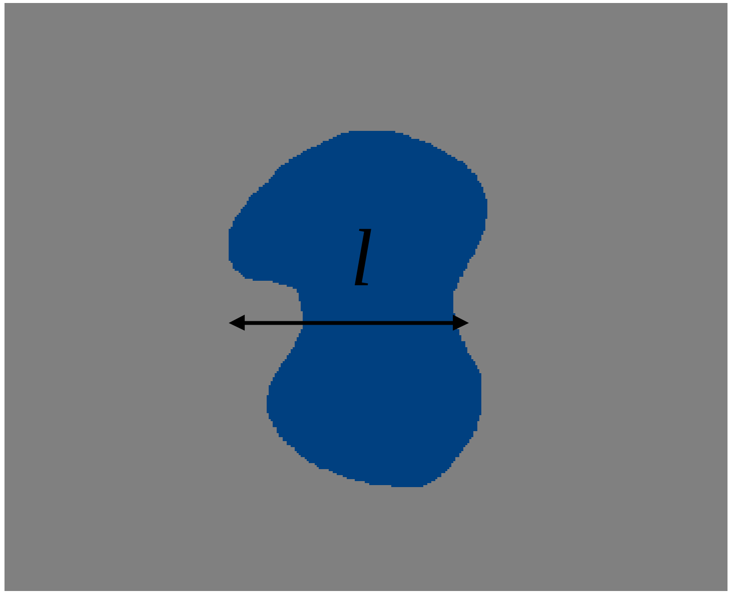
$$\lim_{H\to 0}\lim_{T\to 0} M(T,H,L) \approx L^{d/2}$$

This leads a scaling relation

$$y_h = y + \frac{d}{2}$$

Droplet Theory

Fisher & Huse:
A typical low-lying excitation of scale l should have the excitation energy proportional to l^{θ}



θ : Droplet Exponent

Distribution of the droplet excitation energy

$$d\varepsilon\, P(\varepsilon, l) = \frac{d\varepsilon}{\Upsilon l^{\theta}} \tilde{P}\left(\frac{\varepsilon}{\Upsilon l^{\theta}}\right)$$

$$\boxed{y = -\theta}$$

Domain Walls

$$E_{\mathrm{wall}}(L) \propto L^{\theta_S}$$

θ_S : Stiffness Exponent

$$\theta_S = -0.281(5)$$

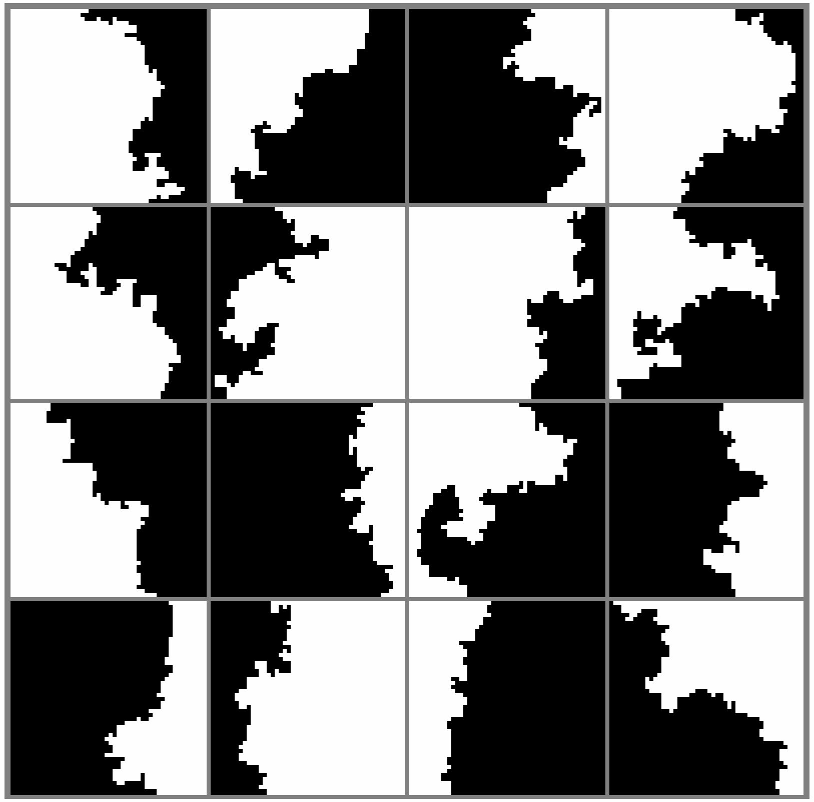


Fig. 1. The domain walls in 16 randomly chosen samples with $L = 48$.

$\theta_S = y$?

Table I. Some of previous estimates of scaling exponents for the two dimensional EA model with the continuous bond distribution. The bond distribution is Gaussian for all the works listed here except for two items with footnotes. $T_{\rm min}$ is the lowest temperature at which the computation for the largest system was performed. In the last column, TM, MC, HO, EO and FF stand for the numerical transfer method (TM), the Monte Carlo simulation (MC), a heuristic optimization (HO), an exact optimization (EO) and mapping to a free fermion problem (FF), respectively

Authors	$-\theta_S$	y_t	$-\theta_D$	Size and Boundary Condition	$T_{\rm min}$	Method
Cheung & McMillan[12)]	0.34(3)[1]			11 (periodic) $\times$ ∞	≥ 0.15	TM
McMillan[7)]	0.281(5)			8 (periodic) $\times$ 8 (p.t.[2])	≥ 0.3	TM
Bray & Moore[5)]	0.291(2)			13 (randomly fixed) $\times$ 12 (periodic)	0	TM
McMillan[13)]	0.306(15)			8 (periodic) $\times$ 8 (periodic)	0	HO
Huse & Morgenstern[10)][3]	0.24(3)[1]			8 (periodic) $\times$ ∞	0	TM
Cieplak & Banavar[14)]	0.31(2)			10 (randomly fixed) $\times$ 11 (periodic)	0	TM
Kawashima & Suzuki[8)]		0.476(5)		20 (periodic) $\times$ 20 (periodic)	0	HO
Kawashima et al.[15)]		0.48(1)		16 (periodic) $\times$ 16 (free)	≥ 0.1	TM
Liang[16)]		0.50(5)		128 (periodic) $\times$ 128 (periodic)	≥ 0.4	MC
Rieger et al.[17)]	0.281(2)			30 (periodic) $\times$ 30 (periodic)	0	EO
Rieger et al.[17)]		0.48(1)		60 (periodic) $\times$ 60 (periodic)	0	EO
Nifle & Young[18)]		0.55(7)		20 (periodic) $\times$ 20 (periodic)	≥ 0.8	MC
Huse and Ko[11)][4]		0.37[5]		40 (p.t.) $\times$ 40 (p.t.)	—	FF
Matsubara et al.[19)]	0.2			21 (free) $\times$ 20 (periodic)	0	MC
Kawashima[9)]			0.47(5)	49 (free) $\times$ 49 (free)	0	HO
present	0.290(10)			48 (free) $\times$ 48 (free)	0	EO

(1) ... The estimate of $1/\nu_{\parallel}$. (See text)
(2) ... Periodic tiling. (See text)
(3) ... The exponential bond distribution as well as the Gaussian distribution was used.
(4) ... The distribution used was symmetric, consists of two continuous parts, and has a vanishing weight around $J \sim 0$.
(5) ... This may not be considered as an estimate of y_t. (See text)

Direct Estimation of θ

Kawashima & Aoki

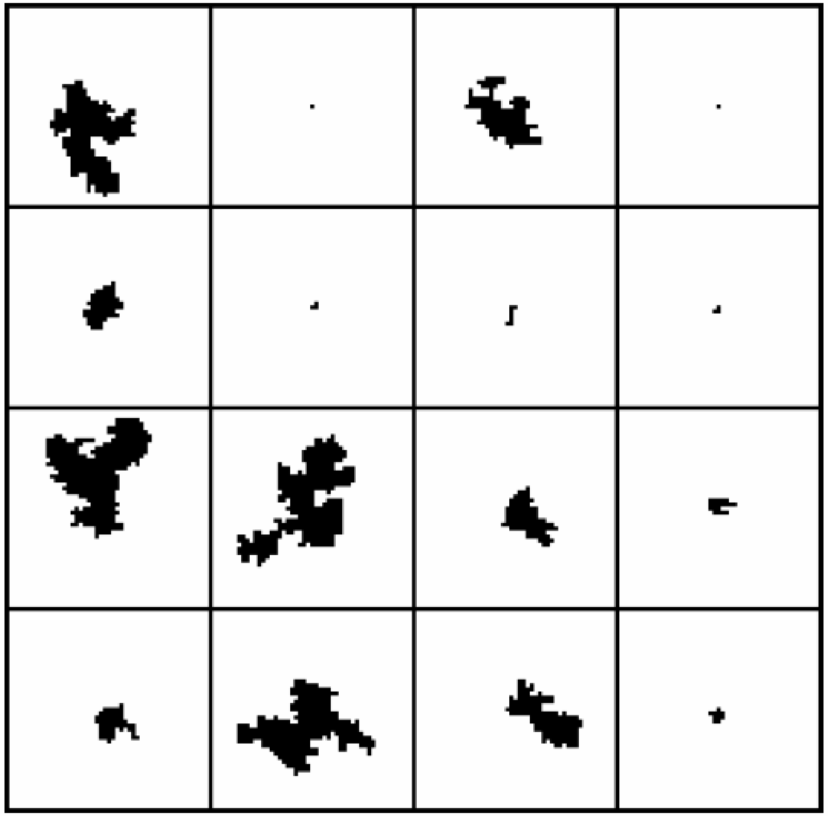
$$V(L) \propto L^{D}$$

$$D = 1.80(2)$$

$$\varepsilon(V) \propto V^{\theta/D}$$

$$\theta = -0.47(5)$$

$$\boxed{\theta_{\mathrm{S}} \neq \theta = -y\,?}$$



$L \leq 49$

Larger Computation

Hartmann & Moore:

$$\varepsilon = Al^{\theta} + Bl^{-\omega}$$

$$\theta = -0.47$$

$$\theta = -0.29$$

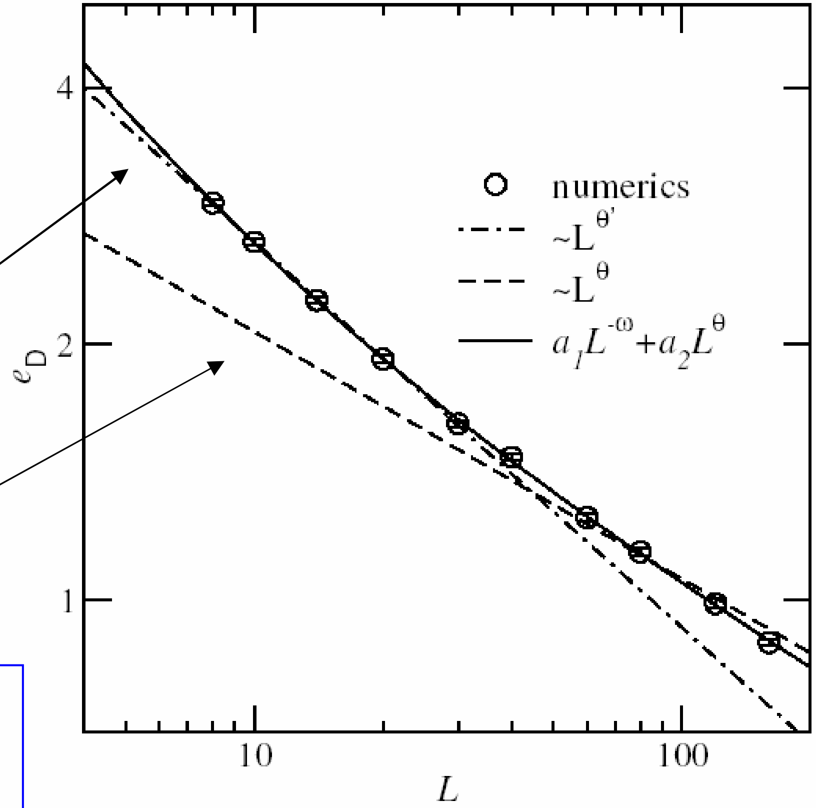


FIG. 1. Average droplet energy E as a function of the system size. The dashed straight lines represent algebraic functions with exponents $\theta' = -0.47$ and $\theta = -0.29$, while the full curved line represents the function from Eq. (2) with $\omega = 1$, $\theta = -0.29$).

Mapping to a linear programming
+ Polynomial-time algorithm
+ LEDA

Still Puzzling…

$$V(L) \propto L^D$$

$$D = 1.80(2)$$

KA's result is confirmed?

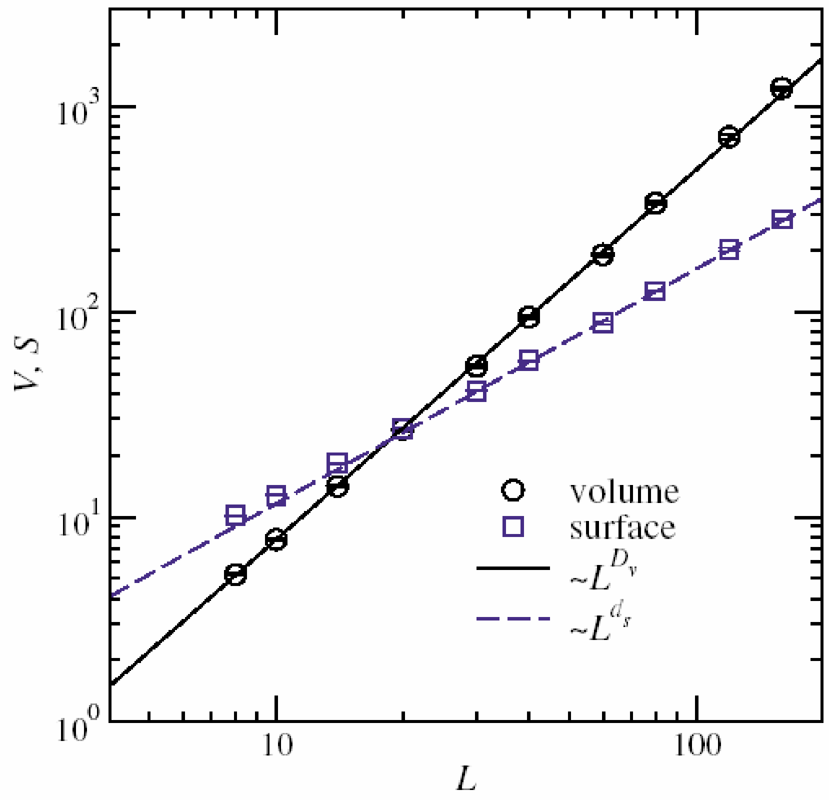


FIG. 2 (color online). Average droplet volume V and average droplet surface S as a function of the system size. The straight lines represent algebraic functions with exponents $D_v = 1.8$, respectively, $d_s = 1.1$.

Lessens to Learn

We have to be very careful in saying something based on a numerical simulation …